**Soft Hesitant Sets**

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| **Abstract**  Fuzzy sets and soft sets are important tools for dealing with uncertainty. There are many extensions of fuzzy sets. The hesitant fuzzy set is one of them. In this study, we define the concept of soft hesitant sets and present some representations of them. We also introduce the set-theoretical operations between two soft hesitant sets and give their examples. |
| Keywords: Soft set, Hesitant Fuzzy set, Soft hesitant sets, set operation |

1. **Introduction**

From past to present, human beings have struggled to cope with situations involving uncertainty, both in daily life and in scientific problems. Many theories have been put forward by scientists to model uncertainty. The most well-known of these is the fuzzy set theory put forward by Zadeh [1] in 1965. Fuzzy sets were introduced as a generalization of classical sets. While in classical set theory, whether or not an element belongs to a set is taken into consideration, in fuzzy set theory, an element may belong to a set somewhat. In a fuzzy set, the degree of belonging of an element to the set is characterized by a function called membership function, which takes values from the closed interval [0,1]. Since its introduction, fuzzy set theory has played an important role in modeling decision-making problems in science, social sciences, medicine and engineering. However, determining the membership function that characterizes the fuzzy set is not always easy. To eliminate this difficulty, soft set theory was proposed by Molodtsov [2] in 1999. Molodtsov defined the soft set as a set-valued function going from the parameter set to the power set of an initial universe. After Molodsov's definition, Maji et al. [3] worked on soft set operations. They defined the union, intersection and complement operations of soft sets and examined the properties of these operations. Ali et al. [4] defined some new soft set operations and examined their properties. Çağman and Enginoğlu [5] redefined soft set operations in a more appropriate way for decision-making problems and applied soft sets in decision making. Sezgin and Atagün [6] studied on soft set operations.

The cocept of hesitant fuzzy sets was introduced by Torra and Narukawa [7] and Torra [8]. While a single membership value corresponds to an element in a fuzzy set, more than one membership value may correspond to an element in fuzzy sets. In this respect, fuzzy sets offer a more effective approach than fuzzy sets in modeling multi-criteria group decision-making problems. By combining hesitant fuzzy sets with soft sets, the concept of hesitant fuzzy soft sets was first introduced by Wang et al.[9]

In this study, considering the difficulty in determining the membership function in fuzzy sets in some cases, we introduce the soft hesitant set structure and set operations between soft hesitant sets with an approach similar to the definition of soft sets without the need for a membership function. Furthermore, we introduce the score function of soft hesitant elements and a notation to facilitate their representation in computer language. This study is an initial study for the mentioned set theory. Therefore, an attempt has been made to make the proposed structure understandable by presenting basic definitions and examples.

1. **Preliminaries**

**Definition 2.1** [7,8] Let be a fixed set, a hesitant fuzzy set (HFS) on is in terms of a function that applied to returns of An is represented as follows:

where is a set of some values in denoting the possible membership degrees of the element to set . is called a hesitant fuzzy (HF) element (HFE).

**Example 2.2** Let be an initial universe. Then, an HFS can be constructed as follows:

Here , , ,

**Definition 2.3** [7,8] Let and be three HFEs. Then, union, intersection and complement of the HFEs are defined as follows:

Union:

Intersection :

Comlement:

**Definition 2.4** [2] Let be an initial universe, be a set of parameters and . Then, a soft set, denoted by , is a mapping defined by

, such that

Here, denotes the power set of , is called the approximation function of the soft set and the value is a set called approximate value of the soft set for all It is worth noting that the sets maybe arbitrary, empty, or have a nonempty intersection. Thus, a soft set over can be represented by the set of ordered pairs.

1. **Soft Hesitant Sets**

In this section, we define a new concept called soft hesitant set by inspiring the definition of HFS and soft set. We also introduce set-theoretical operations between the soft hesitant sets.

**Definition 3.1.** Let be an initial universe, be a set of parameters and . Then, a soft hesitant set (SHS) is a mapping defined as follows:

such that

Here, is power set of power set of is called approximation function of the soft hesitant set and the value is a subset of called approximate value of the soft set for all An SHS over can be represented by the set of ordered pairs.

From now on, denotes the parameter set, denotes the set of alternatives and set of all SHSs on related to parameter set is denoted by

**Example 3.2** Let be a set of parameters and be the set of house for rent in the real estate agent. Four friends want to rent a house. After the real estate agent shows the available houses, an SHS is created as follows:

Then, SHS is given as follows:

**Definition 3.3** Let Then, if for all , it is called empty SHS, and it is denoted by .

**Definition 3.4** Let Then, if for all , it is called universal SHS, and it is denoted by .

Now, we represent an SHS as a set of digits as follows:

Let be a set of parameters and be ordered parameter sequence of the parameters in . Let be an initial universe and be ordered elements sequence in . -approximate element is defined as follows:

()

Here and

**Note:** is maksimum number of SH elements in any If number of SH elements in any is , then arrays (0,0,…,0) are added to .

**Example 3.5** Let us consider SHS given in Example 3.2, Then,

**Definition 3.6** Let  be an SHS over and parameter set . Then score vector of alternatives, denoted by is obtained as follows:

Here denotes the score of the

**Example 3.7** Let us consider the SHS given in Example 3.2. Then,

and maximum score is .

**Definition 3.8** Let and be two SHSs over Then, union of and , denoted by , is defined as follows. For

Here .

**Definition 3.9** Let and be two SHSs over Then, intersection of and , denoted by , is defined as follows: For

.

Here, .

**Definition 3.10** Let be an SHS over . Then, complement of , denoted by , is defined as follows:

Here .

**Example 3.11** Let be a set of parameters and be the set of alternatives. Let and be two SHSs given as follows:

Then,

}

1. **Conclusion**

In this study, a useful set structure in group decision-making problems is defined. Basic definitions and set operations regarding the flexible hesitant set are given without going too much into the theoretical parts. In later stages, algebraic and topological structures can be studied on this cluster structure. We also believe that it will offer researchers a perspective in modeling decision-making problems in various fields.

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