**On Intuitionistic Fuzzy Soft Multisets**

***Fatih Karamaz1,\*[C:\Users\Abdullah\AppData\Local\Microsoft\Windows\INetCache\Content.Word\ORCID-iD_icon-16x16.gif](https://orcid.org/0000-0002-4184-1848), Faruk Karaaslan1[C:\Users\Abdullah\AppData\Local\Microsoft\Windows\INetCache\Content.Word\ORCID-iD_icon-16x16.gif](https://orcid.org/0000-0002-0836-6264)***

*1Department of Mathematics, Faculty of Sciences, Çankırı Karatekin University, Çankırı, Turkey*

|  |
| --- |
| **Abstract**  An intuitionistic fuzzy set is characterized by membership and non-membership functions. The notions of the soft set and multiset are other useful instruments in the modeling of some problems. In this paper, the concept of intuitionistic fuzzy soft multisets (IFSMSs) and set-theoretical operations of IFSMSs are defined. Fundamental definitions and operations are supported with examples to make the concepts more understandable. |
| Keywords: Intuitionistic fuzzy set, Soft sets, Intuitionistic fuzzy soft set, Multiset, Intuitionistic fuzzy soft multisets |

1. **Introduction**

In real life, human being encounters some situations involving uncertainty and incomplete information. Researchers seek constantly new ways to cope with such situations. As a result of this search, new mathematical theories have emerged. The well-knowns of these are fuzzy set theory [1], intuitionistic fuzzy set theory [2], rough set theory [3], soft set theory [4], neutrosophic set theory [5], and multiset theory [6]. Recently, the properties and applications of the intuitionistic fuzzy sets and multisets have been studied extensively.

The concept of the soft set was introduced by Molodtsov [4] as a mathematical tool to deal with some uncertainties. Then, Maji et al. [7] defined several operations between two soft sets and derived some properties of soft set operations. The soft set theory has a rich application area in solving practical problems in economics, social science, medical science etc. Many interesting results of soft set theory have been studied by embedding the ideas of fuzzy sets and intuitionistic fuzzy set.

In the solving of the decision-making problems multisets, intuitionistic fuzzy sets, and soft sets have individual advantages. In this paper, to utilize by integrating the advantages of mentioned sets in the modeling of the decision-making problems, we define the concept of intuitionistic fuzzy soft multiset. we define the concept of intuitionistic fuzzy soft multisets (IFSMSs). We also define the set theoretical operations of IFSMSs and obtain some of their properties.

1. **Preliminaries**

In this section, some definitions and operations which they will be required in the next sections are given.

**Definition 2.1. [1]** Let be a nonempty set called initial universe. Then, a fuzzy set is defined by its membership function as follows:

the value is called the membership degree of . This numerical value expresses the degree of belonging to the fuzzy set . Also, fuzzy set on can be written as follows:

.

From now onward will be denote the family of all fuzzy sets .

**Definition 2.2. [8]** Let be a nonempty set, be an indexing set and a family of partially ordered sets. A fuzzy multiset in is a set:

.

**Definition 2.3. [2]** An intuitionistic fuzzy set (IFS) over is defined as an object of the following form   
 where the functions and define the degree of membership and the degree of non-membership of the element , respectively, and for every .

In addition, for all , , are intuitionistic fuzzy universal and intuitionistic fuzzy empty set, respectively.

**Definition 2.4. [2]** For every two IFS’s and the following operations and relations are valid:

1. **Inclusion:**
2. **Equality:** and
3. **Complement:**
4. **Intersection:**
5. **Union:**
6. **Addition:**
7. **Multiplication:**

**Definition 2.5. [9]** Let A be an IFS. Then, is called the score function of intuitionistic fuzzy elements of A.

**Definition 2.6. [10]** A triangular norm (t-norm, for short) is a map which satisfies:

,

,

,

.

**Definition 2.7. [10]** A triangular norm (t-conorm, for short) is a map which satisfies:

,

,

,

.

**Definition 2.8. [10]** For two intuitionistic fuzzy sets and in , we define the generalised intersection and union as: ,

.

where denotes a t-norm and a t-conorm.

**Definition 2.9. [11]** Let X be a nonempty set. An Intuitionistic Fuzzy Multiset A denoted by IFMS drawn from X is characterized by two functions: 'count membership' of and 'count non membership' of given respectively by and where Q is the set of all crisp multisets drawn from the unit interval such that for each , the membership sequence is defined as a decreasingly ordered sequence of elements in which is denoted by where and the corresponding non membership sequence will be denoted by such that for every and .

An IFMS A is denoted by .

**Definition 2.10. [4,7,12]** Let Let be a set of parameters, and be the universal set of objects. A soft set over is a set of pairs defined as follows: where and is called approximate function of the soft set . The image of under mapping is called -element.

After this time, the set of all soft sets over and parameter sets will be denoted by .

**Definition 2.11.[12]** For every two soft sets and the following operations and relations are valid:

1. **Inclusion:**
2. **Equality:**
3. **Complement:**
4. **Intersection:** for all
5. **Union:** for all

**Definition 2.12. [13,14]** Let be an initial universe, be the set of all intuitionistic fuzzy set over , be a set of all parameters and . Then an intuitionistic fuzzy soft set (IFS-set) over is a function from into .

Where, the value is an intuitionistic fuzzy set over . That is, , where and are the membership and non-membership degrees of to the parameter , respectively.

Note that, the set of all intuitionistic fuzzy soft sets over is denoted by .

**3. Intuitionistic Fuzzy Soft Multisets**

Let be a universal set and be a parameter set. Then, an intuitionistic fuzzy soft multiset (IFSMS) is a set defined as follows:

such that for . In generally, such elements are not displayed in IFSMS. Here, is called intuitionistic fuzzy multi element (IFME) and n denotes number of different intuitionistic fuzzy element (IFE) in IFME corresponding to .

Here, denotes the repeating number of IFEs in evaluated with parameter .

**Example 3.1.** Consider universal set and parameter set . Then, an IFSMS can be written as

**Definition 3.1.** Let be an IFSMS on and parameter set . Then, cardinality of according to parameter denoted by is defined as follows: .

From now on set of all IFSMS over and parameter set will be denoted by . Throughout the paper, IFMS will be denoted by and IFME will be denoted by .

If , then .

**Definition 3.2.** Let  . Then, and , decreasing sequence of the elements in  according to score function of IFE is denoted by . Here denotes IFEs such that . This sequence is called ordered sequence of . Also, denotes sequence of repetition number corresponding to elements of the ordered sequence .

After that, is called ordered IFSMS (OIFSMS) and is deneoted by . Set of all OIFSMSs on and is denoted by .

**Definition 3.3.**  Let ,   
. Then, it is said to be is subset of , if for , and denoted by .

**Remark 3.1.** Sometimes it may be or . If , and are equalized by adding times to . Set of all equalized OIFSMSs on and is denoted by . Then, subsethood relation is checked. If , then subsethood relation doesn't exist.

**Note 3.1.** Equalization method mentioned above will be used in other set operations such as union, intersection, and so on.

**Example 3.2.** Let us consider the universal set and parameter set . Assume that s and are given as follows:

and

Since, for all , subset , i.e, .

**Definition 3.4.** Let . If and , , then it is called an empty IFSMS, and denoted by .

**Definition 3.5.** Let . If and , , then it is called an universal IFSMS, and denoted by .

**Definition 3.6.** Let . If and , , then it is said that equal to and denoted by .

**Proposition 3.1.** Let . Then

1. .

*Proof.* The proofs are clear from Definition 3.3.

**Corollary 3.1.** Let . Then,

1. .

**Definition 3.7.** Let  . Then the complement of denoted by , is defined as follows:

**Example 3.3.** Consider universal set and parameter set . Assume that IFSMS is given as follows:

Then, the complement of   can be written as

Proposition 3.2. Let . Then,

1. =

*Proof.* The proofs are clear from Definition 3.7.

**Definition 3.8.** Let ,   
. Intersection of and , denoted by , is defined as follows:

**Proposition 3.3.** Let ,, and . Then,

1. =

*Proof.* The proofs are clear from Definition 3.8.

**Definition 3.9.** Let ,   
. Union of and , denoted by , is defined as follows:

**Proposition 3.3.** Let ,, and . Then,

1. =

*Proof.* The proofs are clear from Definition 3.9.

**Example 3.4.** Let us consider the universal set and parameter set . Assume that s and are given as follows:

and

Then

and

.

**Remark 3.2.** Let . If or , then and .

**Acknowledgment**

In this study, the financial support was provided by The Scientific and Technological Research Council of Turkey (TUBITAK), 2211/A - Domestic Doctorate Scholarship Program. The author would like to thanks to all supporters due to their precious contributions.

**References**

1. Zadeh, L. A. (1965). Fuzzy Sets, *Information and Control,* 8, 338-353.
2. Atanassov, K. (1986). Intuitionistic fuzzy sets, *Fuzzy Set and Systems*, 20, 87-96.
3. Pawlak, Z. (1982). Rough sets. *Int J Comput Inform Sci*, 11(5), 341-356.
4. Molodtsov, D. A. (1999). Soft set theory-First results, *Computer and Mathematics with Applications*, 37, 19-31.
5. Smarandache, F. (1999). *A unifying feld in logics. Neutrosophy: neutrosophic probability, set and logic*, American Research Press, Rehoboth.
6. Yager, R. R. (1986). On the theory of bags, *Int. J. General Systems*, 13, 23-37.
7. Maji, P.K., Biswas, R., & Roy, A.R. (2003). Soft set theory. *Computers and Mathematics with Applications*, 41(4), 555-562.
8. Miyamoto, S. (2001). Fuzzy Multisets and Their Generalizations. In: Calude, C.S., PĂun, G., Rozenberg, G., Salomaa, A. (eds) Multiset Processing. WMC 2000. Lecture Notes in Computer Science, vol 2235. Springer, Berlin, Heidelberg.
9. Zhang, Z., Yang, J., Ye, Y., Hu, Y., & Zhang, Q. (2013). Intuitionistic fuzzy soft set theory and its decision making, *Journal of Intelligent Fuzzy Systems*, 24(4), 829-836.
10. Deschrijver, D., & Kerre, E. (2002). A generalisation of operators on intuitionistic fuzzy sets using triangular norms and conorms. Notes on Intuitionistic Fuzzy Sets. 1.
11. Shinoj, T.K., & Sunil, J.J. (2012). Intuitionistic fuzzy multisets and its application in medical diagnosis, *International Journal of Mathematical and Computational Sciences*, 6, 34-37.
12. Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni-int decision making. *European J. Oper. Res.*, 207, 848-855.
13. Maji, P. K., Biswas, R., & Roy, A. R. (2001) Intuitionistic fuzzy soft sets, *J. Fuzzy Math.*, 9(3) 677-692.
14. Çağman, N., & Karataş, S. (2013). Intuitionistic fuzzy soft set theory and its decision making, *Journal of Intelligent Fuzzy Systems*, 24(4), 829-836.