**r,s,t-Spherical fuzzy soft sets**

***Fatih KARAMAZ1,[[1]](#footnote-1)\* [C:\Users\Abdullah\AppData\Local\Microsoft\Windows\INetCache\Content.Word\ORCID-iD_icon-16x16.gif](https://orcid.org/0000-0002-4184-1848), Faruk KARAASLAN1 [C:\Users\Abdullah\AppData\Local\Microsoft\Windows\INetCache\Content.Word\ORCID-iD_icon-16x16.gif](https://orcid.org/0000-0002-0836-6264)***

*1* *Department of Mathematics, Faculty of Sciences, Çankırı Karatekin University, 18100, Çankırı, Turkey*

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| **Abstract**  This study introduces the concept of r,s,t-Spherical Fuzzy Soft Sets (rst-SFSS), building upon and extending the traditional T-Spherical Fuzzy Soft Sets (T-SFSS) framework. The proposed rst-SFSS model enhances the flexibility and applicability of fuzzy soft sets by incorporating distinct degrees for membership, neutral membership, and non-membership, allowing for a more comprehensive representation of uncertain and imprecise information. Within this extended framework, various fundamental operations such as complement, subsethood, equality, union, and intersection are rigorously defined, providing a foundational structure for complex data analysis. Additionally, the properties of these operations are thoroughly examined to establish their logical coherence and practical applications, particularly in fields that rely on multi-criteria decision-making processes. |
| Keywords: r,s,t-spherical fuzzy sets, soft set, fuzzy soft set, r,s,t-spherical fuzzy soft sets |

1. **Introduction**

Fuzzy set theory, established by Zadeh [1] in 1965, has proven to be an essential instrument for addressing uncertainty and ambiguity across diverse scientific and technical domains. Numerous generalizations of fuzzy sets have been proposed over the years to improve their applicability and tackle more intricate decision-making challenges. Prominent among these are Intuitionistic Fuzzy Sets (IFS) [2], Pythagorean Fuzzy Sets (PyFS) [3], Picture Fuzzy Sets (PFS) [4], and Spherical Fuzzy Sets (SFS) [5], each offering different ways to addressing degrees of membership, non-membership, and hesitation.

The notion of Spherical Fuzzy Sets (SFS) [5] was established to offer a more adaptable and thorough framework by integrating degrees of membership, neutral membership, and non-membership within a spherical context. This development facilitated the emergence of T-Spherical Fuzzy Sets (T-SFS) [6], which further expanded the model by employing a parameter n to regulate the total of the degrees.

Building on these developments, this paper introduces the r,s,t-Spherical Fuzzy Soft Set (rst-SFSS), a novel extension that incorporates three parameters r, s, and t to provide an even more flexible and powerful tool for decision-making. The rst-SFSS framework allows for a more nuanced representation of uncertainty and offers improved capabilities for handling multi-criteria decision-making (MCDM) problems.

This study introduces the concept of r,s,t-Spherical Fuzzy Soft Sets (rst-SFSS), expanding on the traditional T-Spherical Fuzzy Soft Sets (T-SFSS) framework. The proposed rst-SFSS model increases the flexibility and applicability of fuzzy soft sets by incorporating distinct levels for membership, neutral membership, and non-membership, enabling a more detailed representation of uncertain and imprecise information. In this extended framework, fundamental operations like complement, subsethood, equality, union, and intersection are rigorously defined, establishing a solid foundation for complex data analysis. Furthermore, the properties of these operations are thoroughly examined to ensure logical consistency and practical application, especially in fields dependent on multi-criteria decision-making processes.

1. **Preliminaries**

This section presents some fundamental principles related to T-SFS and r,s,t-SFS, which are well-documented in the literature.

**Definition 2.1.** [1] Let be a nonempty set called initial universe. A fuzzy set (FS) is denoted by and is defined by its membership function as follows:

the value of is called the membership degree of . This numerical value expresses belonging the degree of to the fuzzy set . Also, fuzzy set on can be written as follows:

The following notions explain the generalization process from IFSs to r,s,t-SFSs.

Let be the universe of discourse with and which are the degrees of membership and non-membership, respectively. The set of is called:

* **Intuitionistic Fuzzy Set (IFS) [2]:** in if it satisfies the condition with the degree of indeterminacy given by
* **Pythagorean Fuzzy Set (PyFS) [3]:** in if it satisfies the condition with the degree of indeterminacy given by

To have further generalization, we consider the universe of discourse with , , and being the degree of membership, degree of neutral membership, and degree of non-membership, respectively. The set is called as follows:

* **Picture Fuzzy Set (PFS) [4]:** in if it satisfies the condition with the degree of refusal given by
* **Spherical Fuzzy Set (SFS) [5]:** in if it satisfies the condition with the degree of refusal given by:
* **T-Spherical fuzzy set (T-SFS) [6]:** Let be any natural number. A set in if it satisfies the condition   
   with the degree of refusal given by:
* **r,s,t-Spherical fuzzy set (r,s,t-SFS) [7,8]:** Let be any natural numbers. A set in if it satisfies the condition with the degree of refusal given by:

Similarly, the extension of Soft Sets to T-Spherical fuzzy soft sets, accompanied with illustrative examples, is documented in the literature, as detailed below.

Let be the universe of discourse and be the set of parameters. The pair is called:

* **Soft Set (SS) [9]** over iff , where is the power set of ;
* **Fuzzy Soft Set (FSS) [10]** over , where is a mapping given by and denotes the set of all fuzzy set of ;
* **Intuitionistic Fuzzy Soft Set (IFSS) [11]** over if and can be represented as:

where represents the set of all IFSs of ;

* **Pythagorean Fuzzy Soft Set (PyFSS) [12]** over if and can be represented as:

where represents the set of all PyFSs of ;

* **Picture Fuzzy Soft Set (PFSS) [4]** over if and can be represented as:

where represents the set of all PFSs of ;

* **Spherical Fuzzy Soft Set (SFSS) [13]** over if and can be represented as:

where represents the set of all SFSs of ;

* **T-Spherical Fuzzy Soft Set (T-SFSS) [14]** over if and can be represented as:

where represents the set of all T-SFSs of .

1. **r,s,t-Spherical Fuzzy Soft Set**

In this section, we introduce r,s,t-Spherical Fuzzy Soft Set (rst-SFSS) as an extension to T-SFSS. Further, the score and accuracy function for the defined rst-SFSS have been proposed along with various operations and different properties.

**Definition 3.1.** Let be the universe of discourse and be the collection of all rst-SFS over . Let be the set of parameters. The pair is a r,s,t-Spherical Fuzzy Soft Set over iff . For any parameter is a r,s,t-Spherical Fuzzy Soft Set given by:

where , , and are the degrees of membership, neutral membership, and non-membership, respectively, with the condition:

and the degree of refusal:

where r,s, and t are natural numbers.

**Example 3.1.** Consider the set of four houses, say, and the set of parameters under consideration,say, . Then, the perception for the attractiveness of the houses may be described as a rst-SFSS given by:

where,

* 1. **Operations on r,s,t-Spherical Fuzzy Soft Sets**

In this subsection, we propose some basic operations on rst-SFSSs.

**Definition 3.2.** Let and be two rst-SFSS on the same universe of discourse . Let be the set of parameters; then:

* **Complement:** where is a mapping given by

;

* **Subsethood:** , iff and for all , ;
* **Equality:** , if and ;
* **Union:** , where for all and

In other words, for all

* **Intersection:** , where for all and

In other words, for all

**Proposition 3.1.** Suppose that and are two rst-SFSSs on the universal set . Let be two subsets of the set of parameters; then, as per their definitions, the following properties clearly hold:

Further, for simplicity and necessary computations, rst-SFSS can also be regarded as and called as r,s,t-Spherical Fuzzy Soft Number (rst-SFSN), where is referential subscript used for establishing a connection between alternatives and parameters in computational examples. For application purposes, to rank these numbers, we propose the score and accuracy functions for the r,s,t-spherical fuzzy soft numbers as follows:

1. **Conclusion**

In this paper, we introduced the r,s,t-Spherical Fuzzy Soft Set (rst-SFSS) as a generalization of the existing T-Spherical Fuzzy Soft Sets (T-SFSS). This new framework enhances the flexibility and applicability of fuzzy set theory in dealing with complex decision-making problems by incorporating the degrees of membership, neutral membership, and non-membership in a more comprehensive manner. We defined various operations on rst-SFSS, including complement, subsethood, equality, union, and intersection, and explored their properties. Additionally, we proposed score and accuracy functions to facilitate the ranking of alternatives, which is crucial in decision-making processes. Furthermore, we developed aggregation operators for rst-SFSS, which play a significant role in the information fusion process. In future research, we aim to explore the potential of rst-SFSS in other domains and develop more sophisticated aggregation operators to further enhance its applicability and efficiency.

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1. \* Corresponding author. *e-mail address: karamaz@karamaz.com* [↑](#footnote-ref-1)