Rough n,m-Rung Orthopair Fuzzy Sets

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Abstract

A rough set approximates a subset of a universal set based on some binary relation and is significant for the reduction of attributes in an information system. An n,m-Rung orthopair fuzzy set provides information about the extent of truthness and falsity of a statement. Both of these theories deal with different forms of uncertainty and can be combined to get their combined benefits. In this paper, we define the concept of rough n,m-Rung orthopair fuzzy sets by combining rough sets and n,m-Rung orthopair fuzzy sets. We also discuss some relationships related to the defined concept. This model can encapsulate two distinct types of uncertainties that appear in imprecise available data through the approximation of n,m-Rung orthopair fuzzy sets in crisp approximation space.

Keywords: Rough n,m-Rung orthopair fuzzy set, n,m-Rung orthopair fuzzy set, Rough set, q-Rung orthopair fuzzy set, Fuzzy set

1 Introduction

Engineering, medicine, social sciences, etc. involve many problems that contain uncertain data. Modelling these problems that involve uncertainty has often been the focus of researchers. However, classical mathematical structures are not sufficient to model these problems. In 1965, Zadeh [1] introduced the concept of fuzzy sets (FSs) to eliminate this weakness of classical sets. In classical set theory, an element is either a member of a set or not. However, in an FS, an element can be a partial member of an FS. An FS is defined by a membership function that maps the elements of a universal set $\mho$ to the interval [0,1]. The membership value (μ) of an element under the membership function indicates the degree of membership of the element to the FS, while 1-(μ) represents the degree of non-membership of the element to the FS. The decision-maker has some hesitancy about membership and non-membership of an element if its non-membership degree is smaller than 1-(μ). In order to model situations that involve hesitancy, Atanassov [9] introduced the concept of intuitionistic fuzzy (IF) sets (IFSs), which is a more general concept than FSs. An IFS is characterized by two functions called membership (μ) and non-membership (ν) functions from a universal set to interval $[0, 1]$. In an IFS, the sum of each element's membership (μ) and non-membership (ν) degree is less than or equal to 1. If the sum of the membership and non-membership degrees is greater than 1, the problem cannot be modelled using IFSs. To overcome this limitation of IFSs, Yager [10] introduced the concept of Pythagorean fuzzy sets (PyFS). In PyFS theory, the sum of the membership and non-membership degrees may be greater than 1, but the sum of their squares must be less than or equal to 1. If the sum of the squares of the membership and non-membership degrees is greater than 1, then another extension of IFSs and PyFSs is required. The concept of q-rung orthopair fuzzy sets (q-ROFSs), which is more useful and effective than IFSs and PyFSs, was defined by Yager [3]. q-ROFS is defined as the sum of the qth powers of membership

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and non-membership degrees being less than or equal to 1. Here, the importance coefficient of membership and non-membership degrees is considered equal. In 2022, Ibrahim and Alshammari [4] introduced the concept of n,m-Rung orthopair fuzzy set, considering that the degrees of membership and non-membership may sometimes not be of equal importance.

The theory of rough sets was presented by Pawlak [2] which is used for the processing and modeling of imprecise data. A rough set is defined on the basis of crisp approximation space comprising a universal set of objects and an equivalence relation defined over this set. The equivalence classes tend to 'granulate' the provided information and the equivalence relation serves as a basic tool to determine Pawlak's rough set. A rough set exhibits imprecision in terms of the boundary of a set. It is represented by a pair of crisp sets, namely lower and upper approximations which are constructed with the help of equivalence relation. The elements in lower and upper approximations are assumed to be surely and possibly contained in the data set, respectively. Many researchers participated in the study of rough set theory [5–8].

Motivated by the following facts, we step ahead in the study of approximate reasoning.

- Rough sets as well as n,m-Rung orthopair fuzzy sets individually deal with different forms of uncertainty. Rough sets deal with the attributes of crisp data whereas n,m-Rung orthopair fuzzy sets work on the characteristics of input through their membership and non-membership functions. Due to the strong complementary relationship between these theories, it is very beneficial to mutually fuse these models for indiscernibility relations and for arbitrary binary relations.

- It is interesting to study information granulation and attribute reduction with the newly proposed model as rough set approximations has a close link with information systems as well as with the quantities and notions associated with them. This approach is significant due to its practical application in the simplification of relevant complex problems as it can minimize attributes and can granulate the available data as well.

Due to the aptness of rough sets and n,m-Rung orthopair fuzzy sets, we propose rough n,m-Rung orthopair fuzzy approximations and discuss their various properties using constructive approach. The main contributions of this study are given below.

- Extending the Pawlak's rough set approximations, this paper presents rough n,m-Rung orthopair fuzzy sets and investigates some identities and properties for lower as well as upper rough n,m-Rung orthopair fuzzy operators.

- The proposed approximation operators are then generalized for arbitrary binary relations which are illustrated through examples.

In the rest of the paper, Section 2 recalls and presents some definitions. Section 3 defines the concept of n,m-Rung orthopair fuzzy set and examines its properties. Section 4 presents conclusions and future work.

2 Preliminaries

In this section, we recall and present the notions of binary relation (BR), Fuzzy Set (FS), Rough Set (RS), q-Rung Orthopair Fuzzy Set (q-ROFS), and n,m-Rung Orthopair Fuzzy Set (nm-ROFS).

Definition 2.1. *A binary relation (BR) ρ from $\mathcal{M}$ to $\mathcal{N}$ is a subset of $\mathcal{M} \times \mathcal{N}$. If it is taken $\mathcal{M} = \mathcal{M}$, then a subset of $\mathcal{M} \times \mathcal{M}$ is named as binary relation on $\mathcal{M}$.*

If ρ is a BR on $\mathcal{M}$ then,

(i) ρ is reflexive if $(m, m) \in \rho$; for all $m \in \mathcal{M}$

(ii) ρ *is symmetric if* $(m_1, m_2) \in \rho$ *implies* $(m_2, m_1) \in \rho$*; for all* $m_1, m_2 \in \mathcal{M}$

(iii) ρ *is transitive if* $(m_1, m_2), (m_2, m_3) \in \rho$ *implies* $(m_1, m_3) \in \rho$*; for all* $m_1, m_2, m_3 \in \mathcal{M}$*.*

If ρ *satisfies conditions (i), (ii), and (iii), then it is called an equivalence relation (ER).*

Definition 2.2. *[1] Let* $\mho$ *be a nonempty set called initial universe. A fuzzy set (FS) is denoted by* Ω *and is defined by its membership function* κ_Ω *as follows:*

$$\kappa_\Omega : \mho \to [0, 1]$$

the value of $\kappa_\Omega(\alpha)$ *is called the membership degree of* $\alpha \in \mho$*. This numerical value* $\kappa_\Omega(\alpha)$ *expresses belonging the degree of* α *to the fuzzy set* Ω*. Also, fuzzy set* Ω *on* $\mho$ *can be written as follows:*

$$\Omega = \{(\alpha, \kappa_\Omega(\alpha)) : \alpha \in \mho, \kappa_\Omega(\alpha) \in [0, 1]\}.$$

Definition 2.3. *[2] Let* $\mho$ *be a finite initial universe and let* $\rho \subseteq \mho \times \mho$ *be an arbitrary equivalence relation over* $\mho$*. Define the equivalence class of* $\alpha \in \mho$ *with respect to relation* ρ *as*

$$[\alpha]_\rho = \{\alpha' \in \mho : (\alpha, \alpha') \in \rho\} \quad \forall \alpha.$$

Then the crisp approximation space is given by the pair $(\mho, \rho)$*. The lower and upper approximations of an arbitrary set* $\mho' \subseteq \mho$ *can be computed as*

$$\underline{\rho}\mho' = \{\alpha \in \mho : \rho(\alpha) \subseteq \mho'\}$$

$$\overline{\rho}\mho' = \{\alpha \in \mho : \rho(\alpha) \cap \mho' \neq \emptyset\}$$

where $\underline{\rho}, \overline{\rho} : \mathcal{P}(\mho) \to \mathcal{P}(\mho)$ *are called lower and upper approximation operators and* $(\underline{\rho}\mho', \overline{\rho}\mho')$ *is called rough set.*

Definition 2.4. *[3] Let* $\mathbb{N}$ *be the set of all natural numbers and* $\mho$ *be a universal set. A q-Rung Orthopair Fuzzy Set (q-ROFS)* ϱ *on* $\mho$ *is defined as follows:*

$$\varrho = \{\langle \alpha, (\mu_\varrho(\alpha), \nu_\varrho(\alpha)) \rangle : \alpha \in \mho\},$$

where $\mu_\varrho : \mho \to [0, 1]$ *and* $\nu_\varrho : \mho \to [0, 1]$ *denote the degree of membership and the degree of non-membership, respectively. Here, the following condition is satisfied for each* $\alpha \in \mho$ *and* $q \in \mathbb{N}$*.*

$$0 \leq \mu_\varrho^q(\alpha) + \nu_\varrho^q(\alpha) \leq 1.$$

Definition 2.5. *[4] Let* $\mathbb{N}$ *be the set of all natural numbers and* $\mho$ *be a universal set. A n,m-Rung Orthopair Fuzzy Set (nm-ROFS)* κ *on* $\mho$ *is defined as follows:*

$$\kappa = \{\langle \alpha, (\mu_\kappa(\alpha), \nu_\kappa(\alpha)) \rangle : \alpha \in \mho\},$$

where $\mu_\kappa : \mho \to [0, 1]$ *and* $\nu_\kappa : \mho \to [0, 1]$ *denote the degree of membership and the degree of non-membership, respectively. Here, the following condition is satisfied:*

$$0 \leq \mu_\kappa^n(\alpha) + \nu_\kappa^m(\alpha) \leq 1$$

for all $\alpha \in \mho$ *and* $n, m \in \mathbb{N}$ *such that* $n \neq m$*. Then, there is a degree of indeterminacy of* $\alpha \in \mho$ *defined by*

$$r_\kappa(\alpha) = \sqrt[n+m]{1 - [\mu_\kappa^n(\alpha) + \nu_\kappa^m(\alpha)]}, \quad r_\kappa(\alpha) \in [0, 1].$$

3 Rough n,m-Rung Orthopair Fuzzy Set

In this section, we define the notion of Rough n,m-Rung Orthopair Fuzzy Sets (RnmROFS) and we obtain some of their properties. Examples are given to make the basic definitions more understandable.

Definition 3.1. *Consider an approximation space $(\mho, \rho)$, where ρ denotes equivalence relation over $\mho$. We can define the lower and upper Rough n,m-Rung Orthopair Fuzzy approximations of a nm-ROFS $\kappa = \{\langle \alpha, \mu_\kappa(\alpha), \nu_\kappa(\alpha)\rangle : \alpha \in \mho\} \in \mathcal{PF}(\mho)$, where $\mathcal{PF}(\mho)$ is the collection of all possible nm-ROFSs over $\mho$, denoted by $\underline{\rho}\kappa$ and $\overline{\rho}\kappa$, as*

$$\underline{\rho}\kappa = \{\langle \alpha, (\underline{\rho}\mu_\kappa(\alpha), \underline{\rho}\nu_\kappa(\alpha))\rangle : \alpha \in \mho\},$$
$$\overline{\rho}\kappa = \{\langle \alpha, (\overline{\rho}\mu_\kappa(\alpha), \overline{\rho}\nu_\kappa(\alpha))\rangle : \alpha \in \mho\},$$

where

$$\underline{\rho}\mu_\kappa(\alpha) = \bigwedge_{\alpha' \in [\alpha]_\rho} \mu_\kappa(\alpha'), \qquad \underline{\rho}\nu_\kappa(\alpha) = \bigvee_{\alpha' \in [\alpha]_\rho} \nu_\kappa(\alpha')$$
$$\overline{\rho}\mu_\kappa(\alpha) = \bigvee_{\alpha' \in [\alpha]_\rho} \mu_\kappa(\alpha'), \qquad \overline{\rho}\nu_\kappa(\alpha) = \bigwedge_{\alpha' \in [\alpha]_\rho} \nu_\kappa(\alpha')$$

and $\overline{\rho}, \underline{\rho} : \mathcal{PF}(\mho) \to \mathcal{PF}(\mho)$ are known as lower and upper Rough n,m-Rung Orthopair Fuzzy approximation operators, respectively. It can be observed that $\underline{\rho}\kappa$ and $\overline{\rho}\kappa$ are nm-ROFSs and the pair $(\underline{\rho}\kappa, \overline{\rho}\kappa)$ denotes Rough n,m-Rung Orthopair Fuzzy Set.

Let $(\mho, \rho)$ be a crisp approximation space and κ be a nm-ROFS. The lower and upper rough n,m-Rung orthopair fuzzy approximation of nm-ROFS κ satisfy the following properties:

- $\underline{\rho}\kappa \subseteq \overline{\rho}\kappa$
- $\underline{\rho}\kappa \subseteq \kappa$
- $\kappa \subseteq \underline{\rho}(\overline{\rho}\kappa)$
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- $\overline{\rho}(\underline{\rho}\kappa) \subseteq \kappa$
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- $\overline{\rho}(\underline{\rho}\kappa) \subseteq \underline{\rho}\kappa$

Example 3.1. *Consider a crisp approximation space $(\mho, \rho)$, where $\mho = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ is universe of discourse and $\rho = \{\{\alpha_1, \alpha_2, \alpha_5\}, \{\alpha_3\}, \{\alpha_4, \alpha_6\}\}$ represents the set of equivalence classes over $\mho$. Further, let $\kappa = \{\langle \alpha_1, (0.5, 0.4)\rangle, \langle \alpha_2, (0.6, 0.7)\rangle, \langle \alpha_3, (0.7, 0.7)\rangle, \langle \alpha_4, (0.8, 0.6)\rangle, \langle \alpha_5, (0.9, 0.3)\rangle, \langle \alpha_6, (0.3, 0.2)\rangle\}$ be a nm-ROFS for n=3 and m=2. We can compute the membership and non-membership degrees of the* *members of lower approximation as*

$$\begin{aligned}
\underline{\rho}\mu_\kappa(\alpha_1), \underline{\rho}\mu_\kappa(\alpha_1) &= \Big(\mu_\kappa(\alpha_1) \wedge \mu_\kappa(\alpha_2) \wedge \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \vee \nu_\kappa(\alpha_2) \vee \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \wedge 0.6 \wedge 0.9, 0.4 \vee 0.7 \vee 0.3) = (0.5, 0.7) \\
\underline{\rho}\mu_\kappa(\alpha_2), \underline{\rho}\mu_\kappa(\alpha_2) &= \Big(\mu_\kappa(\alpha_1) \wedge \mu_\kappa(\alpha_2) \wedge \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \vee \nu_\kappa(\alpha_2) \vee \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \wedge 0.6 \wedge 0.9, 0.4 \vee 0.7 \vee 0.3) = (0.5, 0.7) \\
\underline{\rho}\mu_\kappa(\alpha_3), \underline{\rho}\mu_\kappa(\alpha_3) &= \Big(\mu_\kappa(\alpha_3), \nu_\kappa(\alpha_3)\Big) = (0.7, 0.7) \\
\underline{\rho}\mu_\kappa(\alpha_4), \underline{\rho}\mu_\kappa(\alpha_4) &= \Big(\mu_\kappa(\alpha_4) \wedge \mu_\kappa(\alpha_6), \nu_\kappa(\alpha_4) \vee \nu_\kappa(\alpha_6)\Big) \\
&= (0.8 \wedge 0.3, 0.6 \vee 0.2) = (0.3, 0.6) \\
\underline{\rho}\mu_\kappa(\alpha_5), \underline{\rho}\mu_\kappa(\alpha_5) &= \Big(\mu_\kappa(\alpha_1) \wedge \mu_\kappa(\alpha_2) \wedge \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \vee \nu_\kappa(\alpha_2) \vee \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \wedge 0.6 \wedge 0.9, 0.4 \vee 0.7 \vee 0.3) = (0.5, 0.7) \\
\underline{\rho}\mu_\kappa(\alpha_6), \underline{\rho}\mu_\kappa(\alpha_6) &= \Big(\mu_\kappa(\alpha_4) \wedge \mu_\kappa(\alpha_6), \nu_\kappa(\alpha_4) \vee \nu_\kappa(\alpha_6)\Big) \\
&= (0.8 \wedge 0.3, 0.6 \vee 0.2) = (0.3, 0.6)
\end{aligned}$$

Consequently, the lower rough n,m-Rung orthopair fuzzy approximation is

$$\underline{\rho}\kappa = \{\langle\alpha_1, (0.5, 0.7)\rangle, \langle\alpha_2, (0.5, 0.7)\rangle, \langle\alpha_3, (0.7, 0.7)\rangle, \langle\alpha_4, (0.3, 0.6)\rangle, \langle\alpha_5, (0.5, 0.7)\rangle, \langle\alpha_6, (0.3, 0.6)\rangle\}$$

Likewise, we have

$$\begin{aligned}
\overline{\rho}\mu_\kappa(\alpha_1), \overline{\rho}\mu_\kappa(\alpha_1) &= \Big(\mu_\kappa(\alpha_1) \vee \mu_\kappa(\alpha_2) \vee \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \wedge \nu_\kappa(\alpha_2) \wedge \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \vee 0.6 \vee 0.9, 0.4 \wedge 0.7 \wedge 0.3) = (0.9, 0.3) \\
\overline{\rho}\mu_\kappa(\alpha_2), \overline{\rho}\mu_\kappa(\alpha_2) &= \Big(\mu_\kappa(\alpha_1) \vee \mu_\kappa(\alpha_2) \vee \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \wedge \nu_\kappa(\alpha_2) \wedge \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \vee 0.6 \vee 0.9, 0.4 \wedge 0.7 \wedge 0.3) = (0.9, 0.3) \\
\overline{\rho}\mu_\kappa(\alpha_3), \overline{\rho}\mu_\kappa(\alpha_3) &= \Big(\mu_\kappa(\alpha_3), \nu_\kappa(\alpha_3)\Big) = (0.7, 0.7) \\
\overline{\rho}\mu_\kappa(\alpha_4), \overline{\rho}\mu_\kappa(\alpha_4) &= \Big(\mu_\kappa(\alpha_4) \vee \mu_\kappa(\alpha_6), \nu_\kappa(\alpha_4) \wedge \nu_\kappa(\alpha_6)\Big) \\
&= (0.8 \vee 0.3, 0.6 \wedge 0.2) = (0.8, 0.2) \\
\overline{\rho}\mu_\kappa(\alpha_5), \overline{\rho}\mu_\kappa(\alpha_5) &= \Big(\mu_\kappa(\alpha_1) \vee \mu_\kappa(\alpha_2) \vee \mu_\kappa(\alpha_5), \nu_\kappa(\alpha_1) \wedge \nu_\kappa(\alpha_2) \wedge \nu_\kappa(\alpha_5)\Big) \\
&= (0.5 \vee 0.6 \vee 0.9, 0.4 \wedge 0.7 \wedge 0.3) = (0.9, 0.3) \\
\overline{\rho}\mu_\kappa(\alpha_6), \overline{\rho}\mu_\kappa(\alpha_6) &= \Big(\mu_\kappa(\alpha_4) \vee \mu_\kappa(\alpha_6), \nu_\kappa(\alpha_4) \wedge \nu_\kappa(\alpha_6)\Big) \\
&= (0.8 \vee 0.3, 0.6 \wedge 0.2) = (0.8, 0.2)
\end{aligned}$$

which gives the following upper rough n,m-Rung orthopair fuzzy approximation

$$\overline{\rho}\kappa = \{\langle\alpha_1, (0.9, 0.3)\rangle, \langle\alpha_2, (0.9, 0.3)\rangle, \langle\alpha_3, (0.7, 0.7)\rangle, \langle\alpha_4, (0.8, 0.2)\rangle, \langle\alpha_5, (0.9, 0.3)\rangle, \langle\alpha_6, (0.8, 0.2)\rangle\}.$$

Thus, $(\underline{\rho}\kappa, \overline{\rho}\kappa)$ is rough n,m-Rung orthopair fuzzy set. Further, one can verify by simple calculations that the above-mentioned properties are also satisfied.

Theorem 3.1. *Let $(\mho, \rho)$ be a crisp approximation space and let $\kappa, \kappa_1, \kappa_2 \in \mathcal{PF}(\mho)$. The lower and upper rough n,m-Rung orthopair fuzzy approximations of κ, κ_1 and κ_2 provides the following properties:*

L1) $\underline{\rho}(\kappa^c) = (\overline{\rho}\kappa)^c$

L2) $\underline{\rho}\mho = \mho$

L3) $\underline{\rho}(\kappa_1 \cap \kappa_2) = \underline{\rho}\kappa_1 \cap \underline{\rho}\kappa_2$

L4) $\underline{\rho}(\kappa_1 \cup \kappa_2) \supseteq \underline{\rho}\kappa_1 \cup \underline{\rho}\kappa_2$

L5) $\kappa_1 \subseteq \kappa_2 \Rightarrow \underline{\rho}\kappa_1 \subseteq \underline{\rho}\kappa_2$

U1) $\overline{\rho}(\kappa^c) = (\underline{\rho}\kappa)^c$

U2) $\overline{\rho}\emptyset = \emptyset$

U3) $\overline{\rho}(\kappa_1 \cup \kappa_2) = \overline{\rho}\kappa_1 \cup \overline{\rho}\kappa_2$

U4) $\overline{\rho}(\kappa_1 \cap \kappa_2) \subseteq \overline{\rho}\kappa_1 \cap \overline{\rho}\kappa_2$

U5) $\kappa_1 \subseteq \kappa_2 \Rightarrow \overline{\rho}\kappa_1 \subseteq \overline{\rho}\kappa_2$

where κ^c denotes the complement of κ.

Proof. The proofs are clear. □

4 Conclusion

The rough n,m-Rung orthopair fuzzy set introduced in this study has the capacity to handle the uncertainties associated with the boundary of a set as well as related to belongingness of its elements. A rough n,m-Rung orthopair fuzzy set is, in fact, a pair of n,m-Rung orthopair fuzzy sets which exhibit uncertainty in the framework of crisp indiscernibility. The proposed model is effective for the case when one has to approximate n,m-Rung orthopair fuzzy input with crisp knowledge. The corresponding lower and upper rough n,m-Rung orthopair fuzzy approximations satisfy all basic properties of rough set. The behavior of these operators with regards to different set operations is also described.

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