**A Comparison Study on five estimation methods for Power Shanker Distribution**

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| **Abstract**  Power Shanker distribution is suggested by Shanker and Shukla [1]. Shanker and Shukla [1] discussed some distributional properties of the Power Shanker distribution. However, there is no comparison study regarding the another estimators of the parameter of the Power Shanker distribution in the literature. Therefore, we provide a new expansion for point estimation of the Power Shanker distribution in this study. This study presents five different methods of estimation, such as maximum likelihood, least-squares, weighted least-squares, Anderson-Darling, and Crámer–von-Mises methods to estimate the parameters of the Power Shanker distribution. It is compared the performances of these estimators via a extensive Monte Carlo simulation study. In the simulation study,5000 repetitions have taken at different sample sizes and parameter settings. We compute average bias and mean square error (MSE) of the parameters of the Power Shanker distribution. It is seen that the average bias and MSE decrease when the sample sizes increase as expected according to the simulation results. Thus, it can be concluded that the estimators provide the procedures of the estimation. |
| Keywords: Shanker distribution, Power Shanker distribution, Point Estimation, Monte Carlo simulation |

1. **Introduction**

Recently, it is seen that many researchers have dealt with parameter estimation for various lifetime distributions. The maximum likelihood method is popularly used to estimate the unknown parameters of the distributions. However, in the last decade, many estimators have been studied by many authors as an alternative to the maximum likelihood method. Some of these papers can be listed as follows: Karakaya and Tanış [2], Karakaya and Tanış [3], Tanış and Karakaya [4], Tanış and Karakaya [5], Tanış [6], Tanış [7].

The main aim of this study is to compare the estimation methods for the Power Shanker distribution.

Shanker distribution is proposed by Shanker [8]. The cumulative distribution function (CDF) and probability denstiy function (PDF) of the Power Shanker distribution are

 (1)

and

, (2)

where  and [8].

Power Shanker distribution is suggested by Shanker and Shukla [1] by using the transformation  in (1)-(2). Then CDF and PDF of the Power Shanker distribution are given by

 (3)

and

, (4)

respectively, where  is a scale parameter,  is a shape parameter and [1]. We briefly denote Power Shanker distribution as in this paper. Shanker and Shukla [1] mentioned that the  distribution is a mixture of  and generalized gamma  distributions. For more details about thedistributions please see [1].

1. **Point Estimation**

In this section, we present five different estimators of the parameters of the  distribution such as maximum likelihood estimator (MLE), least squares estimator (LSE), weighted least squares estimator (WLSE), Anderson-Darling estimator (ADE), and Cramér–von Mises estimator (CvME).

Let  be a random sample from the  distribution and  denote the corresponding order statistics. In addition,  denotes the observed value of . The log-likelihood function of the distribution is

 (3)

where  is a parameter vector. Then, maximum likelihood estimator (MLE) of  is given as follows:

 (4)

Let us define the following four functions which are used to obtain the different type of estimates:



and



the LSEs, WLSEs, CvMEs, and ADEs of the parameters  are given, respectively, by

 (5)

 (6)

 (7)

 (8)

The estimators given in (4)-(9) can be obtained by **optim ()** function in R with Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. The BFGS algoritm is firstly studied by Fletcher [9].

1. **Simulation Study**

In this section, we provide a comprehensive Monte Carlo simulation study to evaluate the performances of five estimators according to biases and MSEs. In the simulation study is performed based on 5000 repetitions. We consider the sample size 25, 50, 100, 250, 500 and four parameter settings as follows:

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**Table 1.** Average biases of MLEs, LSEs, WLSEs, ADEs, and CvMEs of  and  parameters

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | | | | |  | | | | |
| **Parameters** | **n** | **MLE** | **LSE** | **WLSE** | **ADE** | **CvME** | **MLE** | **LSE** | **WLSE** | **ADE** | **CvME** |
|  | 50 | 0,0882 | 0,0531 | 0,0670 | 0,0649 | 0,0682 | 0,0459 | 0,0528 | 0,0503 | 0,0522 | 0,0438 |
|  | 100 | 0,0834 | 0,0558 | 0,0695 | 0,0647 | 0,0632 | 0,0465 | 0,0493 | 0,0472 | 0,0500 | 0,0448 |
|  | 250 | 0,0801 | 0,0561 | 0,0703 | 0,0633 | 0,0591 | 0,0472 | 0,0474 | 0,0456 | 0,0493 | 0,0456 |
|  | 500 | 0,0789 | 0,0568 | 0,0708 | 0,0632 | 0,0583 | 0,0471 | 0,0460 | 0,0445 | 0,0484 | 0,0450 |
|  | 50 | 0,0253 | -0,0079 | 0,0006 | 0,0041 | 0,0270 | 0,0041 | 0,0112 | 0,0093 | 0,0089 | 0,0053 |
|  | 100 | 0,0130 | -0,0040 | 0,0023 | 0,0022 | 0,0132 | 0,0011 | 0,0048 | 0,0034 | 0,0036 | 0,0018 |
|  | 250 | 0,0051 | -0,0018 | 0,0013 | 0,0006 | 0,0050 | -0,0006 | 0,0007 | 0,0001 | 0,0004 | -0,0005 |
|  | 500 | 0,0028 | -0,0007 | 0,0013 | 0,0007 | 0,0027 |  | 0,0009 | 0,0004 | 0,0005 | 0,0002 |
|  | 50 | 0,0603 | 0,0004 | 0,0152 | 0,0210 | 0,0644 | 0,0473 | 0,0187 | 0,0248 | 0,0286 | 0,0597 |
|  | 100 | 0,0302 | 0,0002 | 0,0111 | 0,0105 | 0,0317 | 0,0233 | 0,0101 | 0,0147 | 0,0145 | 0,0295 |
|  | 250 | 0,0076 | -0,0044 | 0,0013 |  | 0,0080 | 0,0087 | 0,0025 | 0,0054 | 0,0049 | 0,0101 |
|  | 500 | 0,0052 | -0,0009 | 0,0022 | 0,0011 | 0,0053 | 0,0061 | 0,0030 | 0,0046 | 0,0041 | 0,0067 |
|  | 50 | 0,1096 | -0,0118 | 0,0218 | 0,0308 | 0,1127 | 0,1579 | 0,0530 | 0,0760 | 0,0819 | 0,2067 |
|  | 100 | 0,0522 | -0,0128 | 0,0105 | 0,0103 | 0,0484 | 0,0691 | 0,0088 | 0,0284 | 0,0282 | 0,0804 |
|  | 250 | 0,0265 | 0,0013 | 0,0125 | 0,0102 | 0,0257 | 0,0284 | 0,0061 | 0,0154 | 0,0134 | 0,0340 |
|  | 500 | 0,0156 | 0,0041 | 0,0097 | 0,0077 | 0,0162 | 0,0176 | 0,0072 | 0,0119 | 0,0099 | 0,0211 |

**Table 2.** Average MSEs of MLEs, LSEs, WLSEs, ADEs, and CvMEs of  and  parameters

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | | | | |  | | | | |
| **Parameters** | **n** | **MLE** | **LSE** | **WLSE** | **ADE** | **CvME** | **MLE** | **LSE** | **WLSE** | **ADE** | **CvME** |
|  | 50 | 0,0111 | 0,0071 | 0,0081 | 0,0078 | 0,0092 | 0,0116 | 0,0126 | 0,0122 | 0,0121 | 0,0121 |
|  | 100 | 0,0085 | 0,0052 | 0,0066 | 0,0060 | 0,0062 | 0,0068 | 0,0073 | 0,0070 | 0,0071 | 0,0069 |
|  | 250 | 0,0071 | 0,0040 | 0,0056 | 0,0047 | 0,0043 | 0,0041 | 0,0042 | 0,0040 | 0,0043 | 0,0041 |
|  | 500 | 0,0065 | 0,0037 | 0,0053 | 0,0044 | 0,0038 | 0,0032 | 0,0031 | 0,0030 | 0,0033 | 0,0031 |
|  | 50 | 0,0167 | 0,0236 | 0,0196 | 0,0180 | 0,0259 | 0,0102 | 0,0106 | 0,0103 | 0,0102 | 0,0110 |
|  | 100 | 0,0080 | 0,0113 | 0,0093 | 0,0089 | 0,0119 | 0,0050 | 0,0051 | 0,0050 | 0,0050 | 0,0052 |
|  | 250 | 0,0030 | 0,0045 | 0,0035 | 0,0035 | 0,0045 | 0,0019 | 0,0020 | 0,0019 | 0,0019 | 0,0020 |
|  | 500 | 0,0015 | 0,0022 | 0,0017 | 0,0017 | 0,0022 | 0,0010 | 0,0010 | 0,0010 | 0,0010 | 0,0010 |
|  | 50 | 0,0599 | 0,0825 | 0,0697 | 0,0636 | 0,0933 | 0,0579 | 0,0691 | 0,0615 | 0,0587 | 0,0819 |
|  | 100 | 0,0281 | 0,0407 | 0,0334 | 0,0316 | 0,0433 | 0,0258 | 0,0318 | 0,0279 | 0,0269 | 0,0347 |
|  | 250 | 0,0102 | 0,0152 | 0,0122 | 0,0120 | 0,0155 | 0,0098 | 0,0119 | 0,0105 | 0,0104 | 0,0123 |
|  | 500 | 0,0051 | 0,0078 | 0,0062 | 0,0061 | 0,0079 | 0,0048 | 0,0060 | 0,0052 | 0,0052 | 0,0061 |
|  | 50 | 0,2428 | 0,3261 | 0,2769 | 0,2546 | 0,3641 | 0,3876 | 0,5552 | 0,4594 | 0,3999 | 0,7113 |
|  | 100 | 0,1136 | 0,1584 | 0,1310 | 0,1253 | 0,1665 | 0,1526 | 0,2157 | 0,1777 | 0,1662 | 0,2420 |
|  | 250 | 0,0418 | 0,0586 | 0,0479 | 0,0472 | 0,0602 | 0,0567 | 0,0831 | 0,0670 | 0,0655 | 0,0871 |
|  | 500 | 0,0205 | 0,0303 | 0,0241 | 0,0238 | 0,0308 | 0,0275 | 0,0418 | 0,0327 | 0,0323 | 0,0429 |

From Tables 1-2, We observe as the sample size increases, the MSEs and biases of all estimators decrease and approach zero. It is clearly seen that the MSEs and biases of the estimators are very close to each other.

1. **Conclusion**

In this study, we compare the estimation methods for the distribution. Firstly, five estimators (MLE, LSE, WLSE, CvME, ADE) of the parameters of the distribution is obtained in point estimation section. Then, a Monte Carlo simulation study is carried out to observe the performances of these estimators according to MSE criterion. As a result of the simulation study, The f MLE is the best estimators according to MSE in Case 2,3 and 4. However, The MSE of LSE of the  parameter is the smallest in Case 1. We recommend the maximum likelihood method and least square method to estimate the parameters of thedistribution according to simulation results.

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